Application No.	

Entrance Examination in 2018
for Admission in October 2018 / April 2019

Department of Fusion Science
School of Physical Sciences
SOKENDAI
(The Graduate University for Advanced Studies)

The five-year PhD course program

Written examination for major subjects

【Instructions】

- Do not open this booklet before being instructed to do so.
- This booklet contains 6 pages, excluding this cover page.
- If you have any problems such as missing or disordered pages or unclear printing, contact the examiners immediately by raising your hand.
- Applicants are to choose 3 questions out of the 6 questions given in this booklet.
- Use the answer sheet (both sides can be used) corresponding to each question.
- Fill in the designated boxes in this booklet and on all 6 answer sheets with your application number.
- Put the symbol "○" in the evaluation request boxes of the answer sheets to be evaluated. And put the symbol "×" in the answer sheets that should NOT be evaluated. If you put the symbol "○" on more than 3 answer sheets, none of the answers will be evaluated.
- The margin of this booklet may be used for memoranda.
- Leave all the pages of this booklet and all answer sheets on the table after the examination.

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<u>Question sheet</u> for the examination
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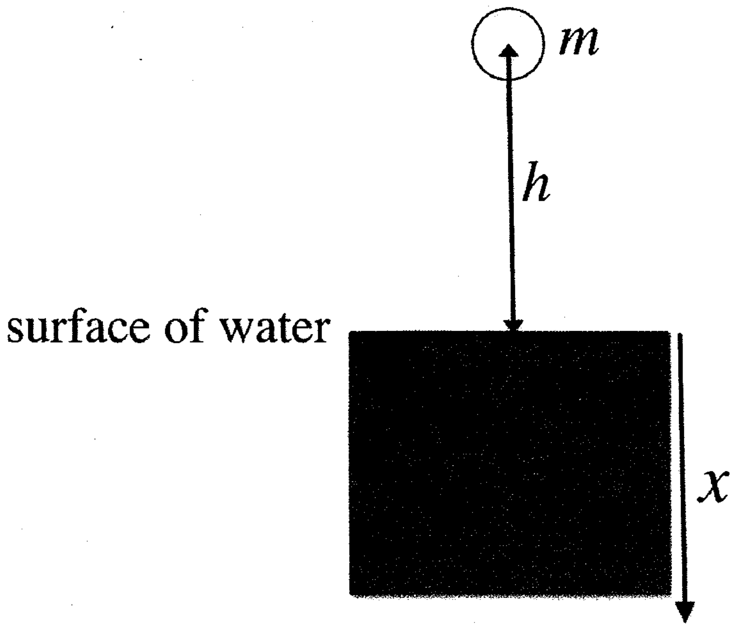
Subject No.	Major subject－ I (classical mechanics)

As shown in the figure below, the object with the mass m freely falls from a height of h with zero initial velocity. Do not consider the size nor shape of the object.

1. Consider the motion until the impact of the object on the surface of water. Write down the equation of motion considering the positive direction as vertical downward. Then, calculate the velocity V_0 on impact with the water. Here, neglect the friction of air. Let the acceleration of gravity be g.

Next, consider the motion in the water. Assume that the buoyant force of the water balances the gravitational force on the object and that the viscous force on the object is bV^2, where b is a positive constant and V is the velocity of the object. Let the depth in the water be x considering the positive direction as shown in the figure. The surface of water is $x = 0$.

2. Write down the equation of motion in the water.
3. Express V as a function of x.
4. Determine the depth of the object, $x(t)$, in terms of time t in the water.



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Subject No.	Major subject－II (electromagnetism)

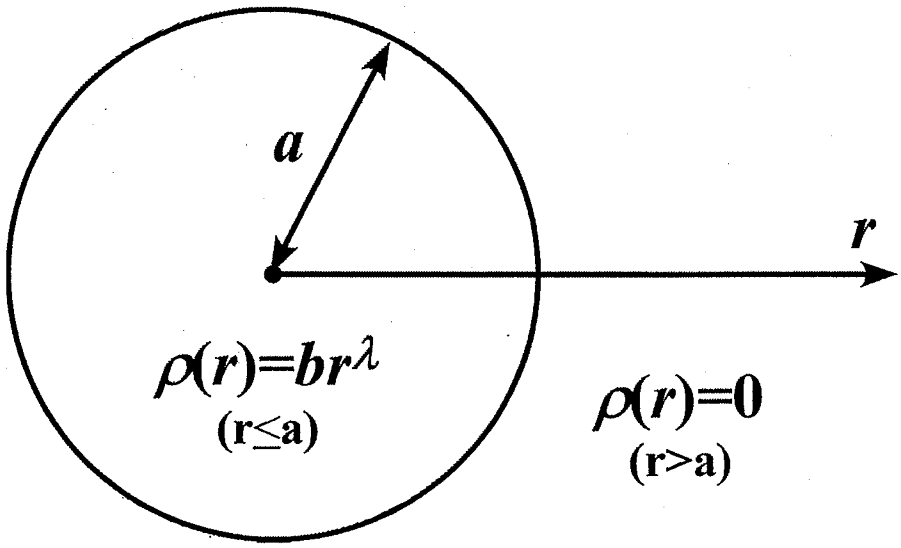
The charge density inside a sphere of radius a $(r \leq a)$ is $\rho(r)=br^{\lambda}$ as shown in the figure, where r is the distance from the sphere center and λ is a nonnegative value. The charge density outside the sphere $(r > a)$ is $\rho(r)=0$.

1. When the total electric charge inside the sphere is Q, obtain b.

2. Using Gauss's law and the result of Question 1, determine the electric field for the two regions $r \leq a$, $r > a$.

3. Determine the electric potential using the results of Question 2.

4. For $\lambda=0$, make a rough plot of the electric field and the electric potential obtained in Questions 2 and 3.



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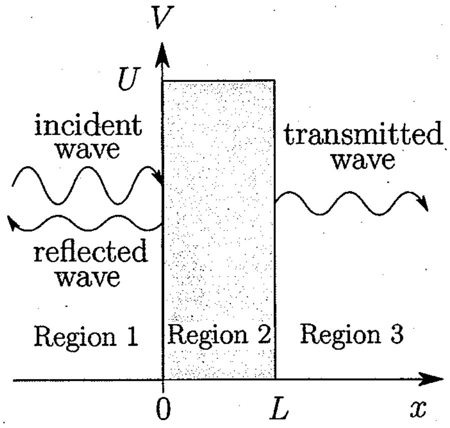
Subject No.	Major subject − III (quantum mechanics)

A particle is incident on a one dimension potential barrier from the left side as shown in the figure. The shape of the potential barrier, $V(x)$, has the width of L and height of U as defined in Eq. (1). The kinetic energy of the particle and the energy of the barrier have a relation such that $0 < E < U$. The particle satisfies a time-independent Schrödinger's equation as shown in Eq. (2). Here, m is the mass of the particle, $\hbar = \dfrac{h}{2\pi}$ and h is Planck's constant. In such a system, some part of the incident wave function is reflected and the rest is transmitted through the barrier. Answer the following questions.



$$V(x) = \begin{cases} 0 & (\text{Region } 1 : x \leq 0) \\ U & (\text{Region } 2 : 0 \leq x \leq L) \\ 0 & (\text{Region } 3 : x \geq L) \end{cases} \quad (1)$$

$$\left[\frac{1}{2m}\left(-i\hbar\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 + V(x)\right]\psi(x) = E\psi(x) \quad (2)$$

1. Modify Schrödinger's equation in Eq.(2) by substituting $k = \dfrac{1}{\hbar}\sqrt{2mE}$ and $\lambda = \dfrac{1}{\hbar}\sqrt{2m(U-E)}$ into the wave functions of $\psi_1(x)$, $\psi_2(x)$ and $\psi_3(x)$ for Regions 1, 2 and 3 defined in Eq. (1), respectively.

2. Find the general solution of the Schrödinger's equations obtained in Question 1 for each Region.

3. The values and the first derivatives of $\psi_1(x)$ and $\psi_2(x)$ should be continuous at $x = 0$, and of $\psi_2(x)$ and $\psi_3(x)$ should be continuous at $x = L$, where $\psi_1(x)$, $\psi_2(x)$ and $\psi_3(x)$ are the wave functions obtained in Question 2. Give the relations of the integral coefficients for Region 1 using those for Region 3 by eliminating the coefficients for Region 2 using the boundary conditions above.

4. The probability-density flux is given by the real part of $\psi^*(x)\dfrac{\hat{p}}{m}\psi(x)$, where $\psi^*(x)$ is the complex conjugate of $\psi(x)$ and $\hat{p}$ is the momentum operator such that $\hat{p} = -i\hbar\dfrac{\mathrm{d}}{\mathrm{d}x}$. Obtain the ratio of the probability-density flux of the transmitted wave function in Region 3, $\psi_3(x)$, to that of the incident wave function in Region 1, $\psi(x)_1$.

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Subject No.	Major subject－Ⅳ (statistical thermodynamics)

Answer the following questions about the thermodynamics of an ideal gas.

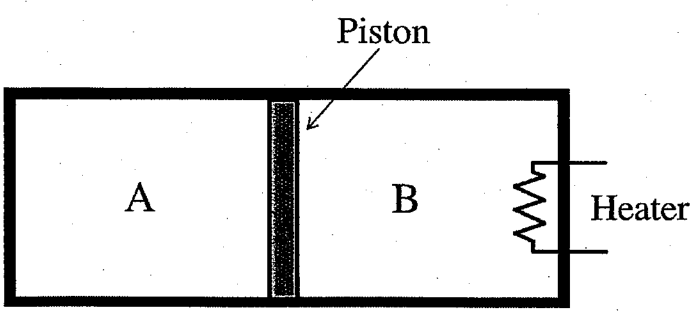
1. Write out the equation of state of an n mol ideal gas. Here, P is the pressure, T is the temperature, V is the volume, and R is the gas constant of the ideal gas.

2. If the ideal gas in Question 1 changes quasi-statically and adiabatically, the relation $nC_V \mathrm{d}T = -P\mathrm{d}V$ is satisfied using the first law of thermodynamics, where $\mathrm{d}T$ is the change of the temperature, $\mathrm{d}V$ is the change of the volume, and C_V is the molar heat capacity at constant volume.
   Derive the relations, $TV^{\gamma-1} = \text{const.}$ and $PV^{\gamma} = \text{const.}$, where $\gamma = \dfrac{C_V + R}{C_V}$.

As shown in the figure, there is a cylindrical vessel which has two chambers, A and B, which are separated by a piston. Here, there are no gap and friction between the vessel and the piston. The vessel and the piston consist of heat insulating materials, and a heater is installed in the chamber B.
As the initial state, a monoatomic ideal gas of 1 mol is filled in each chamber with the same volume V_0, pressure P_0, and temperature T_0.

3. After the gas in the chamber B is heated quasi-statically by the heater, the volume of the gas in the chamber B is changed to V_{B}, and the pressures in both chambers are changed to P_1. In this case, calculate the ratio of the temperature T_{A} in the chamber A to the temperature T_{B} in the chamber B. And calculate the ratio of the pressure P_1 after the heating to the initial pressure P_0 using the relation for an adiabatic process, $PV^{\gamma} = \text{const.}$, in Question 2.

4. In the case that $V_{\mathrm{B}} = \alpha V_0$, $(1 < \alpha < 2)$, calculate the heat Q, which is given by the heater to the gas in the chamber B, using R, T_0, α and γ.
   Here, $C_V = \dfrac{3}{2}R$.



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Subject No.	Major subject — V (mathematics)

Consider a matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 5 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

Answer the questions:

1. Calculate the inverse matrix A^{-1} of the matrix A.

2. Calculate the eigenvalue and eigenvector of the matrix A. Here, you do not have to normalize the eigenvector.

3. Using the eigenvector of the matrix A obtained in Question 2, calculate the regular matrix P and diagonalize the matrix A using P.

4. Here, λ_i is the eigenvalue and $\mathbf{e}_i$ is the normalized eigenvector obtained in Question 2. Prove the theorem of spectral decomposition as follows;

$$A = \sum_{i=1}^{p} \lambda_i \mathbf{e}_i{}^T \mathbf{e}_i,$$

   where p is the number of eigenvalues.

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Subject No.	Major subject－VI (electrical engineering)

Answer the following questions about the two circuits coupled with the mutual inductance, M, as shown in the figure below, where L, i_1, i_2 and C are the self inductance, current in the circuit 1, current in the circuit 2, and static capacitance, respectively. The voltage is given as $E = E_0 \sin(\omega t)$, where t and ω are the time and the angular frequency, respectively.

1. Write a circuit equation for each of the two circuits.

2. Determine i_1 and i_2 by solving the circuit equations in Question 1.

3. Determine the condition for i_2 to be 0.

4. Determine $i_1(t)$ and $i_2(t)$ in the case that $L = 0.1$ H, $M = 0.09$ H, $C = 0.00012$ F, $\omega = 300$ rad/s and $E_0 = 10$ V. In addition, draw a freehand graph of $i_1(t)$ and $i_2(t)$ for $0 \leq t \leq \frac{2\pi}{\omega}$.