Application No.	

Entrance Examination
for Admission in April 2020 / October 2020

Department of Fusion Science
School of Physical Sciences
The Graduate University for Advanced Studies, SOKENDAI

The five-year PhD course program

Written examination for major subjects

【Instructions】

- Do not open this booklet before being instructed to do so.
- This booklet contains 6 pages, excluding this cover page.
- If you have any problems such as missing or disordered pages or unclear printing, contact the examiners immediately by raising your hand.
- Applicants are to choose 3 questions out of the 6 questions given in this booklet.
- Use the answer sheet (both sides can be used) corresponding to each question.
- Fill in the designated boxes in this booklet and on all 6 answer sheets with your application number.
- Put the word "**YES**" in the evaluation request boxes of the answer sheets to be evaluated. And put the word "**NO**" in the evaluation request boxes of the answer sheets that should NOT be evaluated. If you put the word "**YES**" on more than 3 answer sheets, none of the answers will be evaluated.
- The margin of this booklet may be used for memoranda.
- Leave all the pages of this booklet and all answer sheets on the table after the examination.

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Question sheet for the examination
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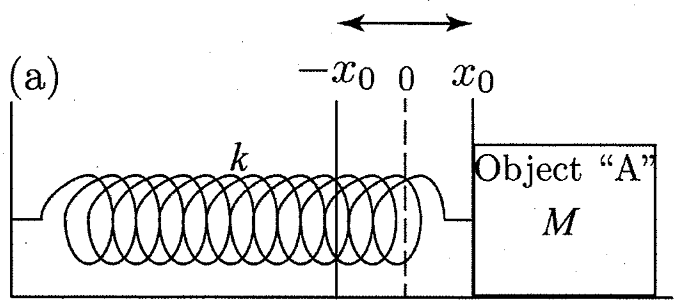
Subject No.	Major subject - I (classical mechanics)

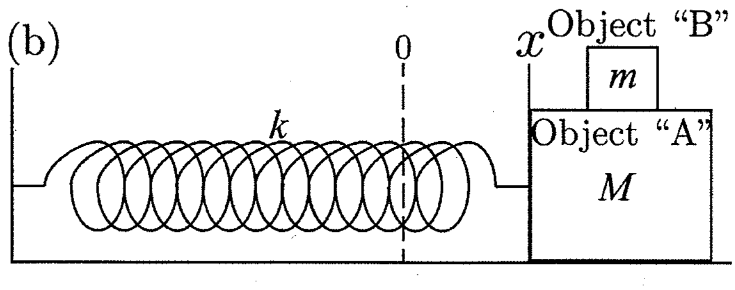
As shown in Figure (a), a solid object "A" with mass M oscillates using a spring on a surface without friction. Let k be the spring constant.

1. Let the direction of motion of object "A" be x. Find the equation of motion of object "A" in the x direction. Here, let the location of the equilibrium length of the spring be $x = 0$.
2. ω is defined as $\omega = \sqrt{\frac{k}{M}}$. At the time $t = 0$, set $x = x_0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$. Express x using x_0, ω and t by solving the equation of motion from Question 1.
3. Let the period of the oscillation of the spring be $\frac{\pi}{6}$ [s]. Set $M = 2$ [kg]. Find the value of k [N/m].

As shown in Figure (b), consider the motion of a solid object "B" with mass $m = 1$ [kg] placed on object "A" using the same spring. A coefficient of static friction of 0.1 exists between objects "A" and "B".

4. When object "A" begins to move after the spring is expanded by a distance x from the equilibrium length, find the maximum value of x for which object "B" does not slip on the surface of object "A". Here, let acceleration of gravity be 9.8 [m/s^2].





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Subject No.	Major subject - II (electromagnetism)

Consider an electromagnet formed by winding a coil of N turns tightly on an unmagnetized iron core shaped like a doughnut with a small gap in vacuum as shown in the figure. The minor and major radii for the doughnut are a and b, respectively, and the width of the gap is δ. The permeabilities of vacuum and the iron core are μ_0 and μ, respectively. A wire of radius r and resistivity ρ is used for the coil. Assume that $r \ll a$ and the length of the wire is $2\pi aN$. The magnet is operated in steady state with a DC power supply of voltage V for the coil. Also, assume that $b \gg a$ and the magnetic flux density is uniform in the core. The edge effect of the gap is ignored. Answer the following questions:

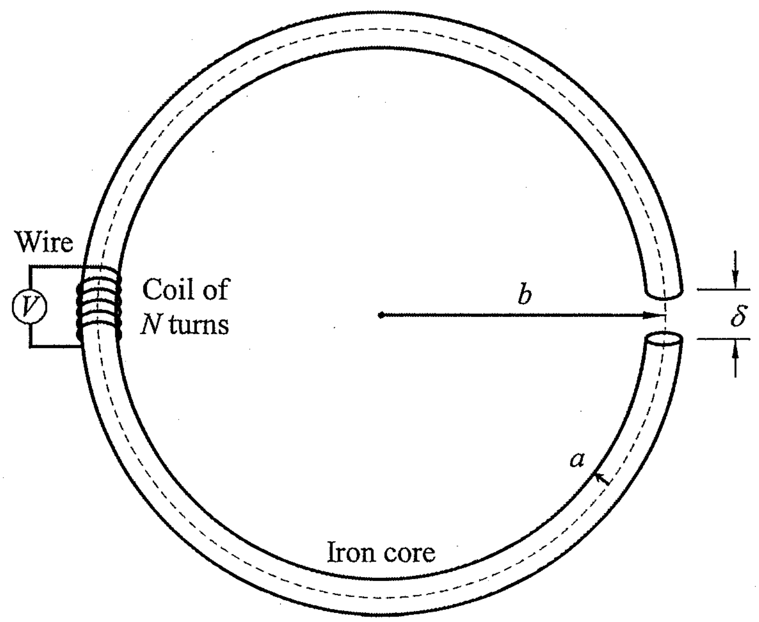
1. Derive the wire resistance R and the wire current I.

2. Derive the power consumed in the coil.

3. Obtain the magnetic flux density B generated in the gap δ of the doughnut at the distance b from the center.

Next, consider when the voltage V is changed abruptly.

4. Obtain the coil self-inductance L and derive the time constant $\tau = \dfrac{L}{R}$ governing the response of the current in the coil.



Subject No.	Major subject - III (quantum mechanics)

For the one-dimensional Schrödinger equation in the x direction for a particle with mass m

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right)\psi(x,t),$$

assume that

$$i\hbar\frac{\partial}{\partial t'}\psi'(x',t') = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x'^2} + V'(x',t')\right)\psi'(x',t') \tag{1}$$

is satisfied under the Galilean transformation, $x' = x - vt$ and $t' = t$. Here, v is a constant, $V(x,t)$ is the potential energy, and $V'(x',t') = V(x,t)$. Furthermore, i is the imaginary unit, and $\hbar = \dfrac{h}{2\pi}$, where h is the Planck constant. Answer the following questions:

1. Write out the differential operators $\dfrac{\partial}{\partial x'}$ and $\dfrac{\partial}{\partial t'}$ in terms of the operators $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial t}$.

2. Assuming that $\psi'(x',t') = e^{iS}\psi(x,t)$, derive the equation for ψ and S by substituting $\psi'(x',t')$ into Eq.(1). Here, S is a non-zero real function of x and t.

3. By setting the coefficients of ψ and $\dfrac{\partial\psi}{\partial x}$ in the results of Question 2 to zero, derive the two equations that $\dfrac{\partial S}{\partial x}$ and $\dfrac{\partial S}{\partial t}$ should satisfy.

4. By solving the equations in Question 3, find $S(x,t)$ such that $e^{iS} = 1$ should be satisfied when $v = 0$.

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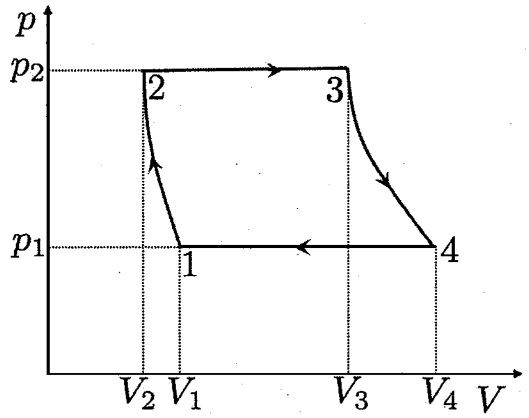
Subject No.	Major subject - IV (statistical thermodynamics)

Consider a quasi-static infinitesimal process of an ideal gas. The right figure shows the Joule cycle, one of the quasi-static circulation processes. There, the change of state 1 → state 2 → state 3 → state 4 → state 1 repeats. The pressure p, the temperature T, and the volume V at states 1, 2, 3, and 4 are (p_1, T_1, V_1), (p_2, T_2, V_2), (p_2, T_3, V_3), (p_1, T_4, V_4), respectively. The processes from state 1 to state 2 and from state 3 to state 4 are adiabatic. Furthermore, the processes from state 2 to state 3 and from state 4 to state 1 are isobaric. The equation of state for an ideal gas is $pV = NkT$, where N is the number of molecules and k is the Boltzmann constant. The constant-pressure specific heat and the constant-volume specific heat are C_p and C_V, respectively, and those are constant. Here, the relation $C_\mathrm{p} = C_\mathrm{V} + Nk$ holds. The ratio of the specific heats, γ is given as $\gamma = \dfrac{C_\mathrm{p}}{C_\mathrm{v}}$.



1. In the adiabatic process of the ideal gas, pV^γ is constant. In the adiabatic process from state 1 to state 2, show that the work $W_{1\to 2} = \int_{V_1}^{V_2} p\mathrm{d}V$ is given by $C_\mathrm{V}(T_1 - T_2)$.

2. Calculate the work $W(= W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1})$ done by the gas using C_p.

3. Calculate the heat quantity Q received by the gas in the process from state 2 to state 3 using C_p.

4. Show that the heat efficiency in this Joule cycle, $\eta = \dfrac{W}{Q}$ is given by

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{\dfrac{\gamma - 1}{\gamma}}.$$

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Subject No.	**Major subject - V (mathematics)**

There is a curve C with a parameter t expressed in Cartesian coordinates as follows:

$$\boldsymbol{r}(t) = \big(x(t), y(t), z(t)\big) = (a\cos t, a\sin t, bt), \ (a \neq 0, b \neq 0).$$

Answer the following questions:

1. Show that the length from $t = 0$ to $t = t_1$ of the curve C is given by $t_1\sqrt{a^2 + b^2}$.
2. Find the unit tangent vector $\boldsymbol{T}$, unit normal vector $\boldsymbol{N}$, and unit binormal vector $\boldsymbol{B}$ of the curve C defined as follows:

$$\boldsymbol{T} = \frac{\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}}{\left|\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right|}, \qquad \boldsymbol{N} = \frac{\frac{\mathrm{d}\boldsymbol{T}}{\mathrm{d}t}}{\left|\frac{\mathrm{d}\boldsymbol{T}}{\mathrm{d}t}\right|}, \qquad \boldsymbol{B} = \boldsymbol{T} \times \boldsymbol{N}.$$

3. Find the equation of the plane that contains the point on the curve C at $t = \frac{\pi}{2}$ such that the unit binormal vector $\boldsymbol{B}$ is perpendicular to the plane at this point.

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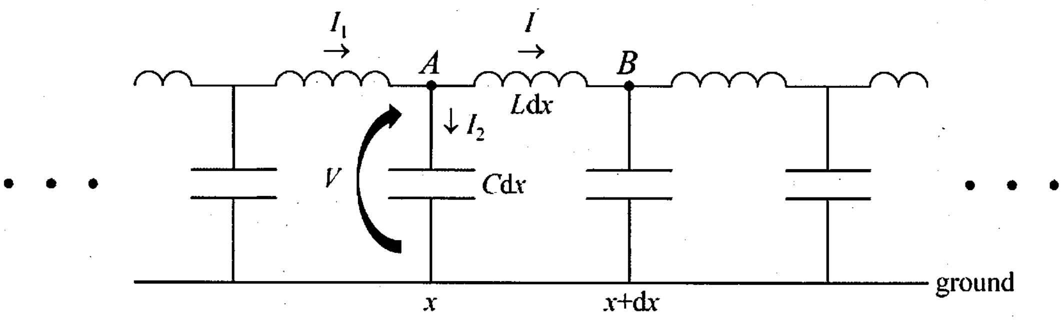
Subject No.	Major subject - VI (electrical engineering)

Consider a transmission line with an infinite number of coils and capacitors whose inductance and capacitance per unit length are L and C, respectively. As shown in the figure below, it is assumed that inductance $L\mathrm{d}x$ and capacitance $C\mathrm{d}x$ are arranged in each infinitesimal section $\mathrm{d}x$. Here, the position of point A along the transmission line is defined as x and the position of point B as $x+\mathrm{d}x$. Answer the following questions assuming that the electric potential at point A relative to the ground is V and the current flowing from point A to point B is I. If necessary, use the formula $\mathrm{d}f(x,t) = \frac{\partial f(x,t)}{\partial x}\mathrm{d}x + \frac{\partial f(x,t)}{\partial t}\mathrm{d}t$ (If $\mathrm{d}t = 0$, then $\mathrm{d}f(x,t) = \frac{\partial f(x,t)}{\partial x}\mathrm{d}x$ and if $\mathrm{d}x = 0$, then $\mathrm{d}f(x,t) = \frac{\partial f(x,t)}{\partial t}\mathrm{d}t$), where t represents time.



1. Prove the following equation by finding the electric potential difference between points A and B due to the inductance:

$$-\frac{\partial V}{\partial x} = L\frac{\partial I}{\partial t}.$$

2. Prove the following equation using the relation that the current I_1 flowing from the left side of point A into point A is the sum of current I and the current I_2 flowing from point A into the bottom side (capacitor side), that is, $I_1 = I + I_2$:

$$-\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}.$$

3. By using the equations proven in Questions 1 and 2 with the relation $\frac{\partial^2 I}{\partial x \partial t} = \frac{\partial^2 I}{\partial t \partial x}$, write the wave equation for the electric potential V. Moreover, by setting $V = V_0 \cos(kx - \omega t)$, find the phase velocity of the wave traveling in the transmission line. Here, k, ω, and V_0 are non-zero real numbers.